Indian Statistical Institute, Bangalore
M. Math.I Year, First Semester
Semestral Examination
Algebra -I (Back Paper)

Time: 3 hours Instructor: N.S.N.Sastry

Maximum Marks 100

Answer all questions. Your arguments should be complete and to the point.

1. Define the tensor product of two modules over a commutative ring R with 1 and specify its R-module structure. If U nd V are vector spaces over $\mathbb{R}$ of dimensions m and n, determine the dimension of their tensor product. [5+10]

2. If M is a module over a commutative ring R with 1, show that $M \otimes R$ is isomorphic to M as an $R-$ module. [8]

3. a) Define the localization of a commutative ring R with 1 at a multiplication subset S of R.

   b) Show that the localization of the ring Z of integers at a prime ideal is neither Noetherian nor Artinian. [8+8]

4. Show that there are only two nonabelian groups of order 8, up to isomorphism. [12]

5. Let p be a prime number. Show that a Sylow p- subgroup of $GL(3, p)$ is a nonabelian group of order p^3. [12]

6. a) Let V be a vector space over a field k. Define the tensor algebra and the symmetric algebra associated with V.

   b) If the dimension of V over k is n , show that the tensor algebra associated with V is isomorphic to the algebra of polynomials in n variable with coefficient in k. [5+5+6]

7. a) Define a projective module over a commutative ring R with 1.

   b) Show that if

$$0 \longrightarrow M' \longrightarrow M \longrightarrow M'' \longrightarrow 0$$

   is an exact sequence of R - modules and P is a projective R - module, then

$$0 \longrightarrow \mathrm{Hom}_R(P, M') \longrightarrow \mathrm{Hom}_R(P, M) \longrightarrow \mathrm{Hom}_R(P, M'') \longrightarrow 0$$

   is also an exact sequence of R - modules. [5+8]

8. Let R be a commutative ring with 1. Define a Noetherian module over R. Show that any finitely generated R - module is Noetherian. [10]

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